S.-T. Yau College Student Mathematics Contests 2016

Oral Exam of Geometry and Topology

Individual

1. Let M be an orientable closed regular surface in the 3-dimensional Euclidean space with positive Gaussian curvature. Prove that the intersection of any two simple (i.e. no self- intersection) closed geodesics on M is non-empty.

2. Let M be a embedded compact surface with positive genus in $\mathbb{R}^3$, show that the Gaussian curvature of M must vanish somewhere on M.

3. Prove the Cartan formulas: $L_X = di_X + i_X d$ and $i_{[X,Y]} = [L_X, i_Y]$.

4.

(1) State Künneth formula for product manifold $M \times N$. Apply it to $S^2 \times S^2$.

(2) Let $f : S^2 \to S^2$ be a degree 2 map. Determine the cohomology defined by the graph of f in $H^*(S^2 \times S^2, \mathbb{Q})$.

(3) Compute the intersection of the graph with the diagonal in $S^2 \times S^2$.